Characterizing a class of social ranking functions

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In Social Choice a problem of great interest is to define a ranking for a set of alternatives, given rankings provided by a fixed number n of agents. Thus, we fix a finite set $A = \{a, b, c, \dots\}$ and a positive integer n; to each alternative $i \in A$ we associate an n-vector $V(i) := (v_1, \dots, v_m)$, $v_s \in I_s$, where I_s, $s = 1, \dots, n$, is the interval $I_s = [\frac{n-s}{n}, \frac{n-s+1}{n}]$. $V(i)$ is called a *valuation of* i and we shall denote by $V_s(i)$ the s-th valuation v_s of i. $T := (V(i))_{i \in A}$ is a set of valuations, one for each alternative; we write $V(i, T)$ whenever we need to specify that the evaluation of the alternative i is part of the set T of evaluations. Finally, denote by $\mathcal{T}(A)$ the set of all evaluations of dimension n over the set of alternatives A. Observe that each single vector of valuations of every alternative is arranged in decreasing order.

Aim of this paper is the study of a class of social ranking Functions. A social ranking solution is a function $F : \mathcal{T}(A) \to \mathcal{R}(A)$. More precisely, our main result provides a small meaningful set of properties, connected to classical ones in Social Choice, to guarantee that the social ranking function is lexicographic. This means that there exists a linear order L on the columns such that in a first step a ranking is made by looking at the evaluations provided by the first column in the order L; since in many cases several alternatives can be indifferent, then the above ranking is refined by looking at the second column in the order L: this can break some of the previous ties. And so on. We further provide an algorithm that, given a lexicographic F, allows finding the linear order L in the above Definition. Finally, we provide a characterization for three specific lexicographic social ranking functions well known in the literature of Social Choice and Voting.

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